FINDING ALL ELEMENTARY CIRCUITS EXPLOITING TRANSCONDUCTANCE

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ABSTRACT

Commonly used elementary circuits like single transistor amplifier stages, the differential pair and current mirror basically exploit the transconductance of transistors. This paper aims at finding ALL elementary transconductance based circuits. For this purpose, all graphs of two-port circuits with one or two Voltage Controlled Current Sources (VCCS) are generated systematically. This leads to 150 graphs. Each of them can be implemented in various ways using transistors and resistors, covering most commonly required types of two-ports. A new Low Noise Amplifier (LNA) implemented in 0.35μm CMOS illustrates the usefulness of the technique.

1. INTRODUCTION

Analog circuit designers commonly use elementary circuits as building blocks for larger circuits. Well-known CMOS circuit examples from textbooks are the common source, common gate and common drain amplifier stages, the CMOS inverter, differential pair and current mirror [1]. Such circuits are for instance cascaded to implement wideband amplifiers, comparators and OPAMPs or synthesize transconductors-C filters. In these circuits, the MOS transistor basically *acts as a transconductor* with transconductance Gm. Therefore we refer to this class of circuits as "Transconductance Based CMOS Circuits" [2]. Important reasons for their use are:

a) MOS transistors behave fundamentally like a transconductor in a wide frequency band. Therefore transconductors are commonly used in RF amplifiers, mixers, oscillators and Gm-C filters.

b) A wide range of transconductance values is possible by changing W/L and biasing. Gm-values may range from nS (weak inversion) up to S (strong inversion, large W/L).

c) Transconductance values can be well-matched (e.g. in the current mirror and differential pair).

d) Transconductance tuning or switching is possible to correct for production spread and aging, or implement programmability (e.g. AGC).

The aim of this paper is to find out whether there are more useful transconductance based circuits than the well-known textbooks circuits. This subject has been addressed in a systematic way in a PhD thesis, generating *ALL two-ports* with 1 and 2 Voltage Controlled Current Sources (VCCS) using linear graphs [2]. This paper subsequently describes: how this was done via graphs (section 2), which types of two-ports can be implemented (section 3), how transistor level circuit implementations are found (section 4), a successful design example of a Low Noise Amplifier (section 5) and conclusions (section 6).

2. GRAPH GENERATION

As discussed in the previous section, we aim to find all elementary transconductance based circuits by using linear graphs. Figure 1 shows conceptually how this is done. As the transconductance is exploited, a VCCS is used as a building block. A four terminal VCCS with isolated voltage and current branch is chosen, as this is a more general and flexible device: it can represent a single NMOS or PMOS transistor (add 1 connection) and a resistor (add 2 connections). Furthermore differential pairs of transistors, or more involved transconductor circuits are covered in this way (no additional connections).

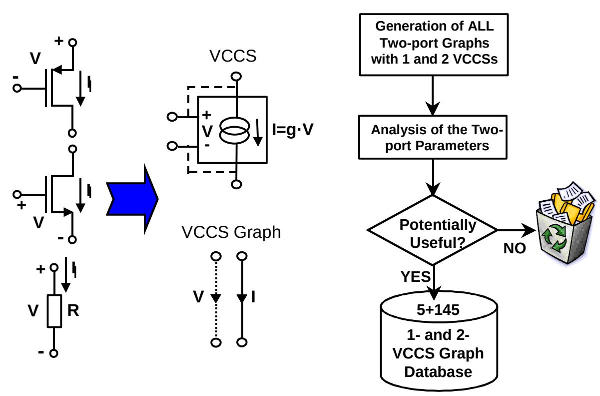
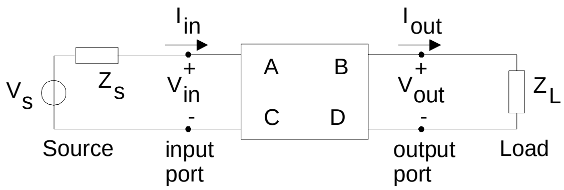


Figure 1: Components in Transconductance based circuits functionally behave as a Voltage Controlled Current Source (VCCS). Its graph representation is a V- and I-branch. All linear graphs of two-ports with 1 and 2 VCCSs have been generated. Potentially useful ones are stored in a VCCS Graph Database.

From a viewpoint of circuit topology a VCCS can be represented by two branches: a V- and I-branch. The aim is now to find all elementary two-port circuits. As shown in Figure 1 this is done by generating all graphs of two-ports with one and two VCCSs. The transfer function of all the two-ports is analyzed, and potentially useful ones are stored in a database. Useful linear two-ports should at least have one non-zero transmission parameter [2] [3], where transmission parameters are defined as shown in Figure 2.



$$A = A(g_a, g_b) \quad A = V_{in} / V_{out} \quad \text{for} \quad I_{out} = 0$$
$$B = B(g_a, g_b) \quad B = V_{in} / I_{out} \quad \text{for} \quad V_{out} = 0$$
$$C = C(g_a, g_b) \quad C = I_{in} / V_{out} \quad \text{for} \quad I_{out} = 0$$
$$D = D(g_a, g_b) \quad D = I_{in} / I_{out} \quad \text{for} \quad V_{out} = 0$$

Figure 2: Two-port described with transmission parameters A, B, C and D. As there are only VCCSs inside the two-port, the transmission parameters are a function of the transconductance g_a and g_b of the VCCSs.

For lack of space we cannot discuss in detail how the graphs were generated. Only the main characteristics of the generation method will be mentioned, starting with the simplest case with one VCCS:

- For one VCCS the graphs consist of 4 branches: the V- and I-branch of the VCCS and a source (S) and load (L) branch.
- Only graphs with all branches connected in loops are useful (otherwise graph branches are not involved in any Kirchhoff relation). So called "tree-graphs" [4] are used as a starting point. "Links" are added to find all undirected unlabelled graphs with 4 branches.
- Graphs are labeled in all different possible ways. Also a (arbitrary) reference direction for the branches is assigned. Reversing V- or I-branch direction only results in a sign change of the corresponding transconductance term in transmission parameter expressions.

Figure 3 shows the resulting useful graphs of two-ports with one VCCS. Apart from the series conductance and parallel conductance, a transconductor can be implemented (e.g. with a common source stage (CS). Also, for large transconductance approximations are possible of a voltage follower (e.g. with a common drain stage (CD)) and current follower (e.g. with common gate stage (CG)).

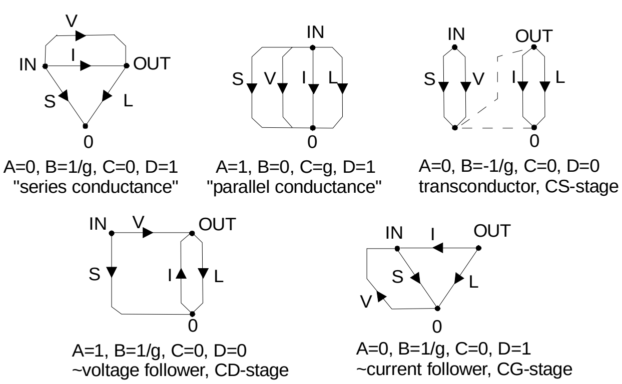


Figure 3: All potentially useful graphs of circuits with a source (S), load (L) and one VCCS (V and I) (see text).

As we are not satisfied with only 5 circuits, an attempt was made to generate all two-ports with 2 VCCSs. As hundreds of graphs exist for this case, computer assistance is very much wanted. From the experience gained in generating the graphs with one VCCS, rules for useless graphs have been inferred and proven. The main simplifying insight gained was that V-branches do not play any role in Kirchhoff current relations (their current is always zero). This enables graph generation in two phases:

1. Generating all "Kirchhoff Current Law graphs" consisting 2 I-branches (from 2 VCCSs) and a source (S) and load (L) branch (no V-branches). This part was still done by hand.
2. Systematically adding V-branches to the graphs in all possible ways and analyzing the transfer function in terms of transmission parameters. This was done by a MAPLE graph generation and symbolic analysis computer program.

This lead to 145 graphs with at least one non-zero transmission parameter. The following section discusses their usefulness.

3. TYPES OF POSSIBLE TWO-PORTS

Unfortunately space does not allow to show all resulting graphs. Only a summary of the results is given here in Table 1 and Table 2. Table 1 shows which combinations of non-zero transmission parameters occur, and how many graphs have this property. The non-zero transmission parameters are a function of the transconductance g_a and g_b of the two VCCSs. The transconductance expressions are of the form:

$$g_i \in \left\{ \pm g_a, \pm g_b, \pm(g_a \pm g_b), \frac{\pm g_a g_b}{g_a \pm g_b} \right\} \qquad (1)$$

They can also be derived in a systematic way from considerations regarding different possible sets of Kirchhoff relations that can be forced in circuits with 2 VCCSs [5].

Case	A	B	C	D	Number
A	1	0	0	0	3
B	0	$1/g_1$	0	0	37
D	0	0	0	1	3
AB	1 or g_1/g_2	$1/g_3$	0	0	24
AD	1	0	0	1	6
BC	0	$1/ g_1$	g_2	0	2
BD	0	$1/ g_1$	0	1 or g_2/g_3	24
ABC	1 or g_1/g_2	$1/ g_3$	g_4	0	3
ABD	1 or g_1/g_2	$1/ g_3$	0	1 or g_4/g_5	24
ACD	1	0	g_1	1	9
BCD	0	$1/ g_1$	g_2	1 or g_3/g_4	3
ABCD	1 or g_1/g_2	$1/g_3$	g_4	1 or g_5/g_6	7

Table 1: The different combinations of non-zero transmission parameters that can be implemented with two VCCSs. Expression g_i is defined in equation (1), where i is an arbitrary index.

Table 2 lists 9 commonly required types of linear two-ports [3], with either very low, very high or accurate port impedances. Using the following expressions for input and output impedance:

$$Z_{in} = \frac{A \cdot Z_l + B}{C \cdot Z_l + D} \quad \text{and} \quad Z_{out} = \frac{B + D \cdot Z_s}{A + C \cdot Z_s} \quad (2a, 2b)$$

it can be derived which non-zero transmission parameters are desired. Table 2 shows them in the third column. The fourth column shows which two-port of Table 1 can be used to implement the desired non-zero transmission parameters. Although there are certainly limitations to what is possible (there are only 2 degrees of freedom via g_a and g_b), it can be concluded that even with only two VCCSs already many useful circuits can be implemented. As might be suspected for VCCS circuits, cases with high and finite port impedances are readily available. For low port impedance, high transconductance is usually needed (except in cases where one VCCS acts as a nullor).

Zin	Zout	Desired	Realizable with case	Additional conditions
∞	0	A	**AB**	B << A·Zl
∞	∞	B	**B**	-
0	0	C	**BC**	B << C·Zs·Zl
0	∞	D	**BD**	B << D·Zs
∞	= Zl	AB	**AB**	-
0	= Zl	CD	**BCD**	B << D·Zs
= Zs	0	AC	**ABC**	B << A·Zl
= Zs	∞	BD	**BD**	-
= Zs	= Zl	ABCD	**ABCD**	A·D = B·C

Table 2: Overview of the implementation possibilities of 9 desired two-ports.

4. FROM GRAPH TO CIRCUIT

Now we have a database of graphs of potentially useful circuits, the question is how to implement them on transistor level. Every VCCS in a graph can in principle be implemented using an arbitrary 4-terminal transconductor implementation. However, a single MOST or even a single resistor can be sufficient in many cases. Whether this is possible depends on the interconnection and the orientation of the V- and I- branch of a VCCS in a graph. Figure 4 illustrates this point. From the figure we see that three different situation occur:

1. If there is *no* connection between the V- and I-branch, a VCCS with a separate floating input and output ports is necessary (right most case). This can be implemented by common source MOST pairs, either of the same or of different type (biasing is not shown).
2. If one connection exists between the V- and I-branch, and the branches have the same orientation (both arrows pointing to or from the common node), a single MOST can be used (a PMOST or NMOST depending on the branch orientation). Alternatively, common source MOST pairs can be used.
3. If the branches are connected at both ends (left-most case), and also have the same orientation, then a simple resistor can be used, apart from single MOSTs or common source MOST pairs.

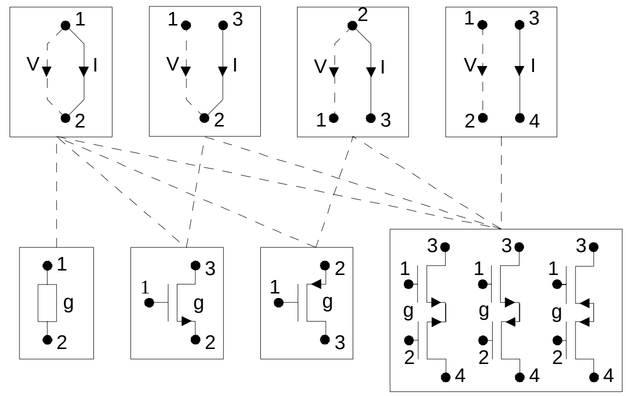


Figure 4: The interconnections between the V- and I-branches, and their relative orientation determine the possibilities to implement a VCCS.

Because of its impact on the implementation possibilities, the direction of the branches will now be examined in more detail. The effect of V- or I-branch reversal is as follows:

- Change the direction of the V- or I-branch: sign change of the related transconductance-term in transmission parameters expressions according to equation (1).
- Change both the direction of the V- and I-branch of the same VCCS: no effect. This transformation changes an NMOST into a PMOST and vice versa. This transformation has no effect on the transfer function, but can be useful for instance for biasing purposes to “fit” circuits in a low supply voltage.

5. CIRCUIT EXAMPLE

To show that this work is not just of academic interest, an example of a practical low noise amplifier design will now be given. Here we focus on the graph generation that was the basis of finding new LNA circuits. Details on the amplifier chip design considerations and measurements can be found elsewhere [6][7]. The main requirements for the LNA were wide bandwidth (50-900MHz for cable modem), input impedance matching to 75ohm, a noise figure <6dB and good linearity (both 2nd and 3rd order). All two-port graphs of amplifiers with input impedance matching and a capacitive load impedance have been selected from the graph database and some additional constraints on stability and wideband operation were posed [6] [7]. The graphs satisfying the requirements are shown in Figure 5. The simplest implementations are shown in Figure 6. Referring to Figure 4 and analyzing the graphs, the simplest implementations of the VCCSs in the graphs are almost always single transistors (case 2 in the previous paragraph). Only g_b in LNA3 can simply be implemented with a resistor (case 3 in the previouse paragraph: parallel branches).

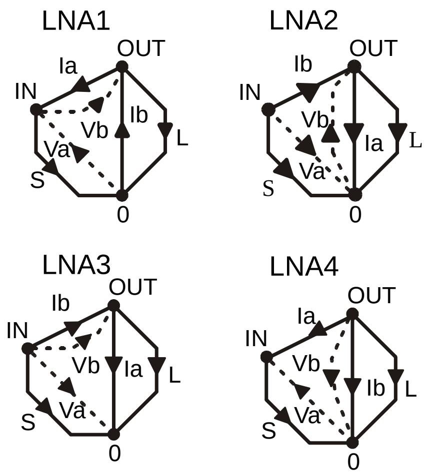


Figure 5: Graphs selected from the VCCS graphs database based on LNA functional requirements.

Two well-known LNAs result (LNA3 and LNA4), but also 2 ones that are new to the best of the authors knowledge (LNA1 and LNA2). LNA1 was realized on chip because of its attractive noise and linearity properties. Table 3 renders the measurement results.

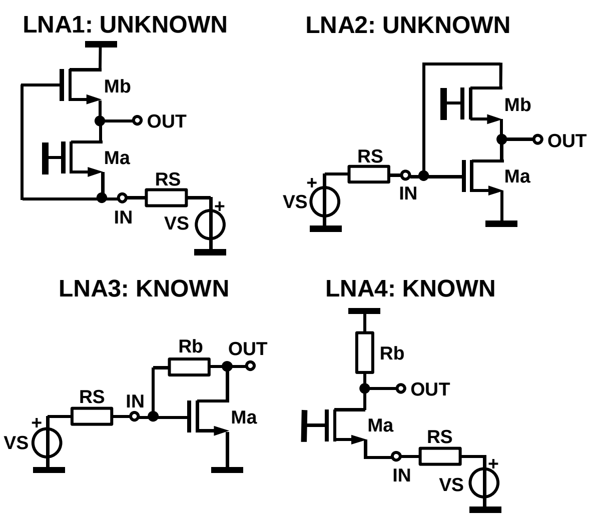


Figure 6: Simplest implementations of the graphs of Figure 5 based on Figure 4 (biasing not shown (voltage bias sources replaced by short-cut to ground, current source by open)).

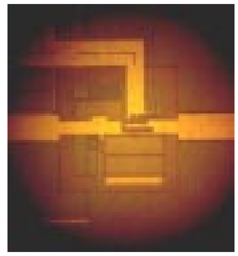
LNA1 Chip realisation	Measured Properties	
	Forward Gain	11 dB
	-3dB bandwidth	900 MHz
	Standing Wave rate (VSWR)	< 1.6
	Reverse Gain	< -30dB
	IIP2 (2nd order Intercept Point)	15dBm
0.35μm CMOS Die area $0.06mm^2$ Supply Voltage 3.3V	IIP3 (3rd order Intercept Point)	1dBm
	Noise Figure	< 4.5dB
	Supply current	1.5mA

Table 3: Experimental results of LNA1 of Figure 6 [6] [7]

6. CONCLUSIONS

ALL elementary transconductance based circuits with one or two VCCSs have been generated systematically using linear graphs, leading to 150 graphs with at least one non-zero transmission parameter. Most commonly required types of two-ports can be implemented based on these graphs. Each VCCS in the graphs can be implemented in various ways, the simplest being a resistor or a single transistor. Hence hundreds of elementary circuits can be found starting from the graphs. The usefulness of the circuit generation is exemplified by a recently published wideband Low Noise Amplifier.

7. REFERENCES

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[7] F. Bruccoleri, E.A.M. Klumperink, B. Nauta, “Generating *ALL* 2-Transistor Circuits Leads to New Wide-Band CMOS LNAs”, Accepted for IEEE Journal on Solid-State Circuits, special issue on ESSCIRC, July 2001.